3D FEM analysis of static fields for nonconforming meshes with node-based, 2nd order elements

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The novel approach of using node-based, second order polynomial shape functions to substitute the degrees of freedom corresponding to slave nodes by a linear combination of those corresponding to master nodes is shown to be a powerful and accurate tool to couple nonconforming meshes. This method is investigated and proposed to be used to take moving domains, especially rotating parts into account. In addition, it is shown that, in comparison to consistent meshes, the number of finite elements can be decreased without any loss of accuracy.

***Index Terms*—Approximation methods, computational electromagnetics, finite element analysis, nonconforming mesh.**

I. Introduction

THE coupling of meshes with different resolution is a known problem in electromagnetic field analysis. A typical application requiring such coupling is the analysis of electric machines with the relative movement of the stator and rotor domains taken into account. One possibility to overcome the problem is to re-mesh the moving domain once the mesh becomes too distorted due to the rotor movement. This method exhibits extensive pre-processing as well as enormous computational costs in addition to the drawback of distorted elements and loss of accuracy [1]. An alternative is to allow the two meshes to become nonconforming with hanging nodes. Then, in case of conducting surfaces, the method of Lagrange multipliers (LM) has been successfully implemented for 2 D problems [1], and for 3 D problems [2],[3]. However, the resulting matrix is not positive definite and ill-conditioned and therefore the equations resulting from the LM method are difficult to solve by iterative techniques like the incomplete Cholesky - conjugate gradient (ICCG) method [4]. The interpolation method by using master and slave edges, has proved its efficiency for first order tetrahedral and hexahedral edge elements [3],[5] and [6].

The novel approach of this paper is to carry out the simple interpolation method for slave nodes by master nodes with node-based, second order polynomial shape functions. The resulting matrix is positive definite, symmetric and the ICCG method can be applied. By a simple electrostatic test problem, the accuracy and the superiority of this method is shown.

II. Nonconforming Mesh Coupling

The Dirichlet-Neumann boundary problem for the electrostatic field in a closed domain Ω is investigated:

$$-div(\varepsilon grad V) = \rho \quad \text{in } \Omega \tag{1a}$$

$$V = U_0 \quad \text{on } \Gamma_E \ , \tag{1b}$$

$$\varepsilon grad V \cdot \mathbf{n} = \varepsilon \frac{\partial V}{\partial n} = \sigma \quad \text{on } \Gamma_D \ . \tag{1c}$$

According to the Dirichlet boundary condition (1b) and taking the properties of the node-based, second order polynomial shape functions N_i into account, it is sufficient to define the approximation to V as $V_n = \sum V_i N_i$ where V_i are the nodal values.

This approach can be taken to introduce the nonconforming mesh connection by defining master nodes m_l and slave nodes s_k on an interface where two meshes become nonconforming, the potential of a slave node $V^{(slave)}$ can be substituted by a linear combination of its corresponding master node potentials $V_l^{(master)}$:

$$V_k^{(slave)} = V(\mathbf{r}_k^{(slave)}) = \sum_{l=1}^{l_{master,k}} c_{kl} V_l^{(master)} \tag{2}$$

where c_{kl} are appropriate coupling factors. Note that these factors are the node-based, second order polynomial shape functions N_l of the corresponding master nodes evaluated at the position of the slave node.

Taking this interpolation into account, the approximation can be written as

$$V_n(\mathbf{r}) = \sum_{l=1}^{l_{normal}} V_l^{(normal)} N_l^{(normal)}(\mathbf{r}) + \\ + \sum_{l=1}^{l_{master}} V_l^{(master)} \left(N_l^{(master)}(\mathbf{r}) + \sum_{k=1}^{l_{slave}} c_{lk} N_k^{(slave)}(\mathbf{r}) \right) \tag{3}$$

where the superscripts stand for the normal (i.e. neither master nor slave), the master and the slave nodes, respectively. This leads to following Ritz-Galerkin equations:

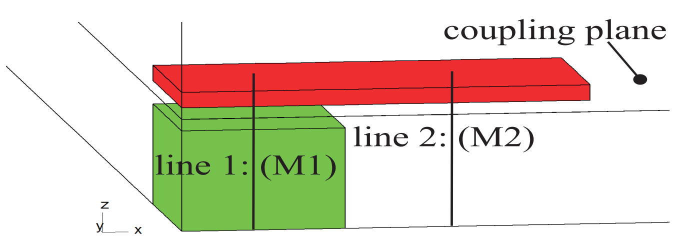
$$\sum_{k=1}^{l_{normal}} V_k^{(normal)} \int_\Omega grad N_i^{(normal)} \ \varepsilon \ grad N_k^{(normal)} d\Omega + \\ + \sum_{k=1}^{l_{master}} V_k^{(master)} \int_\Omega grad N_i^{(master)} \ \varepsilon \ \left(grad N_k^{(master)} + \right. \\ \left. + \sum_{l=1}^{l_{slave}} c_{kl} \ grad N_l^{(slave)} \right) d\Omega \ = \ 0 \ , \tag{4}$$

$$
\sum_{k=1}^{l_{normal}} V_k^{(normal)} \int_\Omega \left(gradN_i^{(master)} + \sum_{j=1}^{l_{slave}} c_{ij}\ gradN_j^{(slave)} \right)
$$
$$
\varepsilon\ gradN_k^{(normal)} d\Omega \ + \ \sum_{k=1}^{l_{master}} V_k^{(master)} \int_\Omega \left(gradN_i^{(master)} + \right.
$$
$$
\left. + \sum_{j=1}^{l_{slave}} c_{ij}\ gradN_j^{(slave)} \right) \varepsilon \left(gradN_k^{(master)} + \right.
$$
$$
\left. + \sum_{l=1}^{l_{slave}} c_{kl}\ gradN_k^{(slave)} \right) d\Omega \ = \ 0\ . \tag{5}
$$

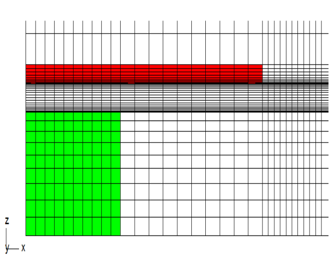
Note that the mixed terms including both master and slave shape functions do not appear, since the support of the two types of shape functions are disjunct.

III. Numerical Application

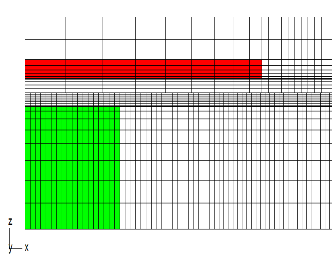
The model of an air insulated parallel capacitor with an inset made of a ceramic medium is investigated, by taking symmetries into account (see Fig. 1(a)). To validate the applied method, both a conforming and a nonconforming mesh model are built. The electrode of the capacitor has the potential of 10 V whereas the $x-y$ plane is set to the potential 5 V. The ceramic medium has a relative permittivity of $\varepsilon_r = 2.4$. The conforming model consisting of 90720 finite elements is shown in Fig. 1(b). Note that, the mesh size is a compromise between the mesh size of the high and low resolution domain of the nonconforming model. Fig. 1(c) shows the nonconforming mesh with a decreased number of 46656 finite elements.



(a)



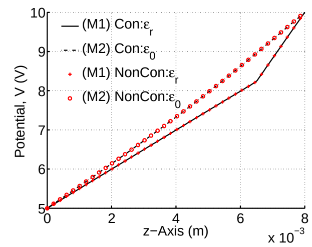
(b)



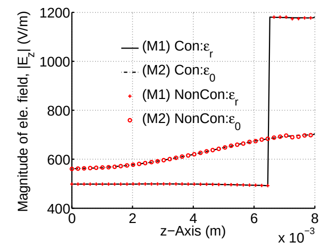
(c)

Fig. 1. (a) 3 D view: electrode (red), ceramic inset (green) and data line ($M1$) and ($M2$), (b) Conforming mesh of the capacitor model, (c) Nonconforming mesh of the capacitor model.

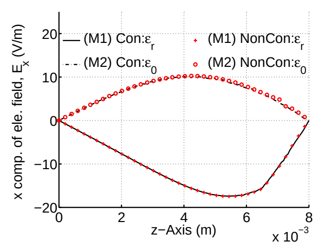
The data shown in Fig. 2 have been taken along two lines parallel to the z-axis (see Fig. 1(a)). Each line has been selected in such a manner that the influence of the edges of the electrode is negligible. The number of iterations of the conjugate gradient (CG) method is 111 for the conforming mesh and 90 for the nonconforming mesh.



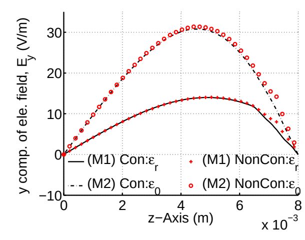
(a)



(b)



(c)



(d)

Fig. 2. (a) Potential V, (b) Magnitude of the z-component of the electric field $|E_z|$ (c) x-component of the electric field E_x, (d) y-component of the electric field E_y

IV. Conclusion and Outlook

As shown in Fig 2, there is a good agreement between the conforming and nonconforming mesh models for all components of the electric field, as well as for the potential. The applied method is seen to be an accurate and powerful tool to decrease the number of finite elements. Numerical experiences show that searching the master/slave coupling needs additional computational resources. Further investigation is needed in this respect. The full paper will include exhaustive accuracy investigations, and the full set of equations for the nonconforming mesh connection method introduced will also be presented.

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